KE-CLOSED SUBCATEGORIES OVER A COMMUTATIVE NOETHERIAN RING

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This talk is based on joint work with S. Saito [KS24, KS25].

Understanding subcategories has been one of the central themes in ring theory. Gabriel [Gab62] achieved a landmark result by classifying Serre subcategories of the category of finitely generated modules over a commutative noetherian ring (or more generally coherent sheaves on a noetherian scheme). Beyond Serre subcategories, much subsequent work has examined other classes of subcategories. One important direction concerns *torsion-free classes* (subcategories closed under subobjects and extensions). For finitely generated modules over a commutative noetherian ring, Takahashi [Tak08] completely classified torsion-free classes in terms of the prime spectrum of the ring.

Motivated by these classification results, we focus on subcategories that are closed under basic categorical operations. Let R be a commutative noetherian ring. Let $\mathsf{mod}\, R$ denote the category of finitely generated R-modules. A subcategory $\mathcal{X}$ of $\mathsf{mod}\, R$ is called *KE-closed* respectively, *CE-closed* or *IE*-closed) if $\mathcal{X}$ is closed under kernels and extensions (respectively, under cokernels and extensions, or under images and extensions). It was recently shown by Stanley–Wang [SW11] and Enomoto [Eno] that CE-closed and IE-closed subcategories coincide with Serre subcategories and torsion-free classes, respectively. In contrast, there exists a KE-closed subcategory which is not a torsion-free class.

The aim of this talk is to introduce our classification of KE-closed subcategories of $\mathsf{mod}\, R$. The outline consists of two parts. First, we associate a function on $\operatorname{Spec} R$ with each KE-closed subcategory of $\mathsf{mod}\, R$, and show that this function completely determines the original subcategory. Second, we investigate when a function arises from KE-closed subcategories. To this end, we introduce the notion of an n-Bass function for each $n \geq 0$, and we establish a bijection between the set of KE-closed subcategories and the set of 2-Bass functions provided that R is (S_2)-excellent in the sense of Česnavičius [Čes21].

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